RN-8047

B. E. - II (Sem. III) (All) Examination

May / June – 2010

Engineering Mathematics - III

Time : 3 Hours] [Total Marks : 100

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. E. - 2 (Sem. 3) (All)**

Name of the Subject :

☛ **Engineering Mathematics - 3**

☛ Subject Code No. : **8 0 4 7** ☛ Section No. (1, 2,.....) : **1&2**

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate marks.
(4) Draw the figure whenever it is necessary.

SECTION - I

Q-1 a) Attempt the following. [10]

1. Define Sturm-Liouville problem & rewrite Legendre's differential equation into Sturm –Liouville problem.
2. Define unit step function and state second shifting theorem.
3. Define duplication formulae & hence evaluate $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$.
4. Solve the initial value problem
   $y'' + y - 6y = 0,\ y(0) = 10\ \&\ y'(0) = 0$
5. Find the value of λ for which the equation
   $(xy^2 + \lambda x^2 y)dx + (x + y)dy = 0$ is exact.

b) Attempt the following [6]

1. Solve $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - x/y\right)dy = 0$.
2. Solve $(x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)xdy = 0$.

c) Define beta & Gamma function & obtain relation between them. **[4]**

Q-2 a) Attempt any two of the following [6]

1. Use method of undetermined coefficient to solve the following BVP
   $y'' + 4y = e^{-x},\ y(0) = 3, y\left(\frac{\pi}{2}\right) = -3$.
2. Solve the following by using method of variation of parameters
   $y'' - 4y' + 5y = e^{2x}\operatorname{cosec} x$.
3. Use method of undetermined coefficient to solve $y'' + 4y = 8x^2$.

b) **Attempt any two of the following** **[6]**

1. Solve $\left(x^2y'' + xy' - y\right) = \frac{1}{x^2}$.
2. Solve $\left(x^2y'' - 2xy' + 2y\right) = x^3 \cos x$.
3. Examine whether $\sqrt{x}, \sqrt{x}\ln x, \sqrt{x}(\ln x)^2 -----, \sqrt{x}(\ln x)^k$ are linearly independent.

c) Use method of reduction to find the second linearly independent solution of $y'' - 4xy' + 4\left(x^2 - 2\right)y = 0;\ y_1(x) = e^{x^2}$. **[3]**

Q-3 a) Obtain a Frobenius series solution of $y'' + x^2y = 0$. **[7]**

OR

Obtain a Frobenius series solution of Legendre's differential equation

$$\left(1-x^2\right)y'' - 2xy' + n(n+1)y = 0,$$

b) **Attempt any two of the following.** **[8]**

1. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.
2. Define Rodrigue's formula & hence derive $P_2(x)$ and $P_3(x)$ from it.
3. Prove that $J_n'(x) + \frac{n}{x}J_n(x) = J_{n-1}(x)$.

SECTION - II

Q-4 a) **Attempt the following.** **[10]**

1. Define Laplace transform & using it find Laplace transform of 1.
2. Define periodic function & state the period of $\sin 2x$.
3. Define Fourier sine & cosine transforms.
4. Define Parsevals identity.
5. State one dimensional wave equation & state which method we apply to find the solution of it.

b) **Attempt any two of the following** **[10]**

1. Find the Fourier series expansion for the function
$$f(x) = \begin{cases} -k \text{ if } -\pi<x<0 \\ k \text{ if } \pi<x<0 \end{cases} \text{ and } f(x+2\pi) = f(x).$$
2. Obtain Fourier series expansion for $f(x) = x\sin x;\ 0<x<2\pi$.
3. Find the Fourier transform of $f(x) = e^{-ax^2}$, *where* $a>0$.

Q-5 a) Define Laplace transformation & using it find Laplace transform of $f(t) = e^{2t}$. **[5]**

b) **Attempt any two of the following.** **[6]**

1. Show that $L(\sinh \omega t) = \frac{\omega}{s^2 - \omega^2}$.
2. Find $L\left(2t + 3e^{2t} + 4\sin 3t\right)$.
3. Find $L^{-1}\left(\frac{4}{(s-4)^3}\right)$.
4. Find the Laplace transform of $t^2 \sin at$.

c) Use convolution theorem to find $L^{-1}\left(\frac{1}{(s-a)(s-b)}\right)$. **[4]**

Q-6 a) Solve the following One-dimensional wave equation **[7]**

$u_{tt} = c^2 u_{xx}; t > 0 \; and \; 0 < x < l$

$u(0,t) = 0 = u(l,t); t > 0$

$u(x,0) = f(x)$

$u_t(x,0) = 0$

Using separation of variables method.

b) Attempt any one of the following. **[8]**

1. Find the deflection $u(x,t)$ of the vibrating string of length π and its ends are fixed, corresponding to zero initial velocity and the initial deflection is $u(x,0) = 2(\sin x + \sin 3x)$.
2. Solve the following boundary value problem

   $u_{xx} + u_{yy} = 0$

   $u(x,0) = u(0,y) = u(l,y) = 0$ and

   $u(x,a) = sin\frac{n\pi x}{l} \; for \; 0 \leq x \leq l, 0 \leq y \leq a$
